On the internal logic of an abstract elementary class

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Abstract

Model theory, and more specifically stability theory, has been generalized in a powerful way to the much wider context of abstract elementary classes (a.e.c.'s). This context includes in a natural way various infinitary logics such as $\mathbb{L}_{\kappa^+,\omega}$, but also several other contexts such as generalized quantifiers or classes with an *ad hoc* "strong extension notion".

Stability theory has been extended successfully over the past decades up to the stable zone for a.e.c.'s; the methods have been predominantly semantic (model constructions, indiscernibles, limit models, minimality, superstability, etc.). There is nonetheless an important "syntactic anchoring" of all this work: Shelah's presentation theorem underscores a good part of these developments. This theorem provides, for each a.e.c. $\mathcal{K}$ in vocabulary τ, an expansion of τ to a larger τ', and in this much larger vocabulary, a way of axiomatizing $\mathcal{K}$ as a *projective* class.

In [4], we give (with Saharon Shelah) a solution to the problem of axiomatizing an a.e.c. in its same vocabulary τ : we provide a sentence $\psi_{\mathcal{K}}$ in the infinitary logic $\mathbb{L}_{\beth_2(\kappa)^{+++},\kappa^+}$, that directly axiomatizes $\mathcal{K}$. We also provide an axiomatization for the $\prec_{\mathcal{K}}$ relation of the a.e.c., as a syntactic generalization of the classical Vaught test from first order, for arbitrary a.e.c.'s.

This also generalizes earlier results due to Kueker [2]; recent results due to Leung go in a very similar direction but with quite different details.

I also plan to speak toward the end on the general problem of capturing the *internal logic of an a.e.c.*. I will draw some comparisons with other recent work with Väänänen [6] in a logic called $\mathbb{L}_{\kappa}^{1,c}$, with some good model-theoretic properties and approximating from below Shelah's $\mathbb{L}_{\kappa}^1$ from [3]. I will present first steps of new joint work with Shelah on the construction of $\mathbb{L}_{aec,\mathcal{K}}$ [5].

References

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