A hierarchy of dependencies

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- to use propositional dependence logic to illustrate two logical phenomena that I call *semantic relativity* and *syntactic sensitivity*.
- to develop a semantics that model these phenomena
- to look at propositional dependence logic through the prism of this semantics

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A uniform account of information

- In logic a piece of information is often understood as a classifier of some semantic objecs (possible worlds in classical logic or some more abstract states in various non-classical logics).
- In most logical systems it is implicitly assumed that there is only one type of information and all indicative sentences convey information of this type.
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- Atomic formulas classify primarily possible worlds.
- Dependence statements (the truth value of q is functionally dependent on the truth value of p) classify primarily sets of possible worlds (teams).

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Principle of semantic relativity

Different kinds of sentences may classify different kinds of semantic objects.

Principle of syntactic sensitivity

The behaviour of logical operators is sensitive to the syntactic features of the statements to which the operators are applied.

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Propositions

- P_n ... The resulting sum is exactly n
- $P_{<n}$ ... The resulting sum is less than n
- $P_{>n}$ ... The resulting sum is greater than n

Fix a possible world: $w = \langle 2, 3 \rangle$
Context of degree 0

Examples of statements that have a truth-value in w:

- P_5,
- $P_{>4}$,
- $\neg P_2$,
- $P_{>3} \wedge P_{<6}$,
- $P_1 \vee P_{>3}$,
- $\neg(P_2 \vee P_3)$.

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Examples of statements that have no truth-value in w:

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Examples of statements that have a truth-value in X:

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Context of degree 2

Examples of statements that have a truth-value in M:

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(Statements of degree 2)

Context and its degree

Definition

- A possible world is a function that assigns to every atomic formula a unique truth value (either T, or F).
- Every possible world will be called a context of degree 0.
- A context of degree $n + 1$ is defined as a nonempty set of contexts of degree n.
- The empty set is called a context of infinite degree.

The degree of a context C will be denoted as $d(C)$.

Language

- $\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi$

The degree of a formula

For every L-formula φ we define the degree of φ, denoted as $d(\varphi)$, in the following way:

- $d(p) = 0$, for every atomic formula p,
- $d(\neg\varphi) = d(\varphi)$,
- $d(\varphi \wedge \psi) = d(\varphi \vee \psi) = max\{d(\varphi), d(\psi)\}$,
- $d(\varphi \to \psi) = max\{d(\varphi) + 1, d(\psi)\}$,

Truth in a context

Now we define a relation of truth $\Vdash$ between contexts and formulas. However, we impose the following restriction:

$C \Vdash \varphi$ is defined if and only if $d(\varphi) \leq d(C)$.

Truth in a context

if $d(\varphi) < d(C)$, then $C \Vdash \varphi$ iff for all $D \in C$, $D \Vdash \varphi$.

Truth in a context

Assume that the degree of the formula on the right is equal to the degree of the context on the left:

$C \Vdash p$ iff $C(p) = T$, for every atomic formula p,

$C \Vdash \neg\varphi$ iff $C \nVdash \varphi$,

$C \Vdash \varphi \wedge \psi$ iff $C \Vdash \varphi$ and $C \Vdash \psi$,

$C \Vdash \varphi \vee \psi$ iff $C \Vdash \varphi$ or $C \Vdash \psi$,

$C \Vdash \varphi \to \psi$ iff $C^{\varphi} \Vdash \psi$,

where

$$C^{\varphi} = \{D \in C \mid D \Vdash \varphi\}.$$

Validity in Logic of Semantic Relativity (LSR)

Definition

$\varphi_1, \ldots, \varphi_n / \psi$ is LSR-valid iff for any context C such that $max\{d(\varphi_1), \ldots, d(\varphi_n), d(\psi)\} \leq d(C)$ if C supports $\varphi_1, \ldots, \varphi_n$ then C supports ψ.

Contradiction, possibility and necessity

- $\bot =_{def} p \wedge \neg p$ $\qquad d(\bot) = 0$
- $\Diamond\varphi =_{def} \neg(\varphi \to \bot)$ $\qquad d(\Diamond\varphi) = d(\varphi) + 1$
- $\Box\varphi =_{def} \neg\Diamond\neg\varphi$ $\qquad d(\Box\varphi) = d(\varphi) + 1$

Dependence

The relation of truth-value dependence *dep*(φ, ψ) is defined as

- $((\varphi \rightarrow \psi) \vee (\varphi \rightarrow \neg\psi)) \wedge ((\neg\varphi \rightarrow \psi) \vee (\neg\varphi \rightarrow \neg\psi))$

$d(dep(\varphi, \psi)) = max\{d(\varphi) + 1, d(\psi)\}$

Variations

- A variation of the formulas $\varphi_1, \ldots, \varphi_n$ is any formula $\chi_1 \wedge \ldots \wedge \chi_n$ where for any i, $\chi_i = \varphi_i$ or $\chi_i = \neg\varphi_i$.
- The set of the variations of $\varphi_1, \ldots, \varphi_n$ will be denoted as $\Sigma(\varphi_1 \ldots \varphi_n)$
- E.g. $\Sigma(p, q) = \{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}$

General definition of dependence

The relation of truth-value dependence *dep*$(\varphi_1, \ldots, \varphi_n, \psi)$ is defined as

- $\bigwedge_{\chi \in \Sigma(\varphi_1, \ldots, \varphi_n)}((\chi \to \psi) \vee (\chi \to \neg\psi))$

$d(dep(\varphi_1, \ldots, \varphi_n, \psi)) = max\{max\{d(\varphi_1), \ldots, d(\varphi_n)\}+1, d(\psi)\}$

Some observations

Proposition

1. $\emptyset \Vdash \varphi$, *for every formula* φ.

Assume $d(C) = d(\varphi) + 1$. *Then*

2. $C \Vdash \Diamond\varphi$ *iff for some* $D \in C$, $D \Vdash \varphi$,
3. $C \Vdash \Box\varphi$ *iff for all* $D \in C$, $D \Vdash \varphi$.

Assume $d(C) = 1$. *Then*

4. $C \Vdash dep(p_1, \ldots, p_n, q)$ *iff for any* $v, w \in C$, *if* $v(p_i) = w(p_i)$, *for any* i, *then* $v(q) = w(q)$.

Failure of substitution

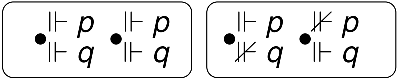
- $q \to r, p \to q / p \to r$ valid
- $q \to \Diamond r, p \to q / p \to \Diamond r$ invalid

(Example: *If Bob is in Munich, he is in Germany. If Bob is in Germany, he might be in Berlin. Therefore, if Bob is in Munich, he might be in Berlin.*)

- $p \to \neg q, q / \neg p$ valid
- $p \to \neg \Diamond q, \Diamond q / \neg p$ invalid

(Example: *If Bob is in Munich, it is not possible that he is in Berlin. It is possible that Bob is in Berlin. Therefore, Bob is not in Munich.*)

A counterexample to Peirce's law: $((p \to q) \to p) \to p$



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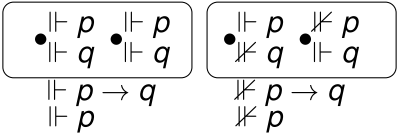

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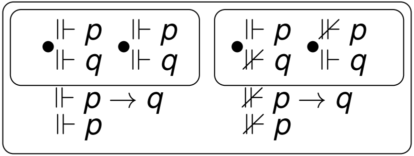

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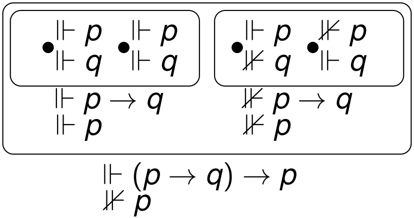
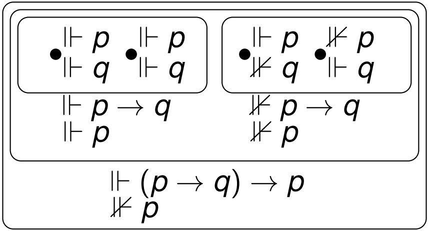


A counterexample to Peirce's law: $((p \to q) \to p) \to p$



$$\nVdash ((p \to q) \to p) \to p$$

Classical fragment

Conditional-free formulas

$$\alpha ::= p \mid \neg\alpha \mid \alpha \wedge \alpha \mid \alpha \vee \alpha$$

Classical formulas

$$\varphi ::= \alpha \mid \alpha \rightarrow \varphi \mid \varphi \wedge \varphi$$

Proposition

For any classical formulas $\varphi_1, \ldots, \varphi_n, \psi$:

- $\varphi_1, \ldots, \varphi_n / \psi$ *is LSR-valid iff* $\varphi_1, \ldots, \varphi_n / \psi$ *is CL-valid.*

Decidability

Proposition

LSR-validity (restricted to finite number of premisses) is decidable.

Functional Completeness

Proposition

Assume that the number of atomic formulas is finite. Then for any number n and any set of n-contexts S, there is an n-formula φ *such that* $||\varphi|| = S$ *(where* $||\varphi||$ *is the set of n-contexts that support* φ*).*

Conclusion

- I have introduce a semantics for the basic propositional language ($\neg, \wedge, \vee, \rightarrow$) that reflects the principles of semantic relativity and syntactic sensitivity.
- The dependence operator can be defined in the basic language and its peculiar properties stem from the two principles.
- Future work: A completeness result