For any $U$, define an equivalence relation $\equiv$. Let $n$ be the set of all finite rooted generated submodels of $\mathbb{N}$. Examples of non-$n$-formulas are formulas containing only $\land$ and $\rightarrow$ as connectives. Visscher, de Jongh, van Bentham and Renardel de Lavalette proved that $n$-formulas are exactly those formulas that are preserved under taking intuitionistic submodels.

NNIL-formulas are sufficient to axiomatize intuitionistic subframe logics, which were earlier axiomatized by M. Zakharyaschev using formulas containing only $\land$ and $\rightarrow$ as connectives.

Universal Models of IPC

The $n$-universal model $U(n)$ for $n$-variable formulas of the intuitionistic propositional calculus (IPC) is isomorphic to the finite depth part of the $n$-Henkin model. $U(1)$ is the Nishimura ladder:

Thm. For any formula $\varphi(p), U(n) \models \varphi$ iff $\varphi(p) \top_{RC} \varphi$.

Thm. For every $w \in U(n) = \langle U(n), R, \vee \rangle$, there exist formulas $\phi_w$ and $\psi_w$ (de Jongh formulas) such that $U(n) \models \phi_w$ iff $wRv$, $U(n) \models \psi_w$ iff $\neg wRv$.

$\equiv$. Let $\mathcal{M} = \langle W, R, V \rangle$ and $\mathcal{N} = \langle W', R', V' \rangle$ be $n$-models. A relation $Z$ on $W \times W'$ is called a subsimulation if it satisfies:

• If $vZw$, then $\mathcal{M}, w \models p_i$ iff $\mathcal{N}, w' \models p_i$, for each $1 \leq i \leq n$.

• If $vZw$ and $vR'w'$, then there exists $w' \in W$ such that $v'Zw'$ and $wRw'$.

We write $\mathcal{M} \preceq \mathcal{N}$ if there exists a total subsimulation $Z$ of $\mathcal{M}$ in $\mathcal{N}$.

Universal Models for NNIL-Formulas

Def. The $n$-universal model $U(n)^f = \langle W, R, V \rangle$ of NNIL($n$)-formulas is defined by taking [Figure to the right: $U(2)^f$]

• $W = S$;

• $[\mathcal{M}]R[\mathcal{M}']$ iff $\mathcal{M} \preceq \mathcal{M}'$ for some $\mathcal{M} \in [\mathcal{M}]$ and $\mathcal{M}' \in [\mathcal{N}]$;

• $\text{color}([\mathcal{M}]) = \text{color}(r_0)$, where $r_0$ is the root of the representative model of $[\mathcal{M}]$.

Note. $U(n)^f$ is finite and rooted.

Thm. For every NNIL($n$)-formula $\varphi$, $U(n)^f \models \varphi$ iff $\varphi \top_{RC} \varphi$.

Thm. For each point $[\mathcal{M}]$ in $U(n)^f$, there exists a NNIL($n$)-formula $\varphi_{[\mathcal{M}]}^f$ such that $[\mathcal{M}] \models \varphi_{[\mathcal{M}]}^f$ iff $[\mathcal{M}]R[\mathcal{M}]$. This work is a part of my master thesis at Institute for Logic, Language and Computation, Amsterdam, The Netherlands.