E-type interpretation without E-type pronoun: How Peirce’s Graphs capture the uniqueness implication of donkey sentences

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1. Introduction

Kadmon (1987, 1990) has made an interesting attempt to explain the uniqueness effects of donkey pronouns under the framework of DRT, by positing some accommodation mechanism that inserts a conditional statement into DRS. It is such conditional statement (denoting scalar implicature, accommodated information, or contextually supplied information) that produces the uniqueness implication associated with donkey pronouns and indefinites. Though successful, there are two problems remaining open. The first problem is that it is not explanatory (circular). In this theory, why accommodation of a conditional statement is needed is due to a felicity condition: donkey pronouns are felicitous iff they refer back to a unique referent. This condition requires that a donkey pronoun anaphoric to a non-c-commanding quantifier a priori carries a uniqueness entailment or presupposition, which is a statement of the facts, not an explanation. The second problem is that it does not explain why discourse anaphora carries a uniqueness implication but syntactic anaphora does not, which also involves variables.

In this paper, I will show that both problems can be satisfactorily answered in a graphic logical system called Existential Graphs (EG), invented by the American philosopher and logician Charles Sanders Peirce (1839-1914). In this logical system, with the help of its inference rules, the uniqueness implication can be derived without a priori assumption of uniqueness residing in the pronouns, and why discourse anaphora carries a uniqueness implication but syntactic anaphora does not follows directly even if both types of pronouns are treated as bound variables.

2. Peirce’s Existential Graphs

The syntax of EG is quite simple with only two logical connectives: conjunction and negation. All graphs (symbols for propositions or predicates, etc) are asserted in a “sheet of assertion” (SA), which “represents the universe of discourse”. To scribe several graphs onto a SA means a relation of conjunction. To assert the falsity of a proposition, a “cut” is to enclose the graph. Other logical connectives are not primitive; they are derivatives. For example, disjunction is defined from de Morgan’s Law: \( \neg(\neg P \land \neg Q) = P \lor Q \). If-then conditional is expressed in two enclosing cuts (scroll), with the antecedent scribing onto the area between the two cuts and the consequent scribing onto the area enclosed by the inner cut.

Quantification is expressed by Line of Identity (LI), which serves as implicit
existential quantification. A line of identity is used to denote both the existence of two objects and the identity between the two objects, if let \( x \) and \( y \) be the variables at the two ends of the line, that is \( x = y \). When the two ends attach to predicate names \( P \) and \( Q \), then the graph \( \overline{P} \) (when the two ends attach to predicate names \( P \) and \( Q \), then the graph \( \overline{P} \) asserts that something which has the property \( P \) exists and it has the property of \( Q \). For example the graph \( \overline{\text{man}_y(y) \text{ran}_x} \) is interpreted as (1).

(1) \( \exists x (\text{man}_x) \land \exists y (\text{ran}_y) \land x = y \)

\[ = \exists x \exists y [\text{man}_x \land \text{ran}_y \land x = y] \]

\[ = \exists y [\text{man}_x \land \text{ran}_x] \]

The default quantifier is existential, represented by a line of identity. Universal quantifier is expressed in a scroll: \( \forall \Leftrightarrow \neg \exists \). A line of identity with evenly enclosed outermost part is read as an existential quantifier; a line of identity with oddly enclosed outermost part is read as a universal quantifier. EG adopts the endoporeutic reading method, an inward-going reading in which one reads with outermost subgraphs or cuts and proceeds inwards. In cases involving two lines, the line of identity first encountered is interpreted first than the line of identity secondly encountered. In other words, the less enclosed the outermost part of a line is, the wider scope the line gets. For example, the sentence Every man loves a woman can have two graphs: each is responsible for a different scope reading.

(2) \( \overline{\text{man}_x(y) \text{loves}_y(y) \text{woman}_y} \)

\[ = \neg[\exists x_1 (\text{man}_x) \land \exists y_2 (\text{loves}_x(y_2) \land \exists y_1 (\text{woman}_y) \land x_1 = y_2 \land y_1 = y_2)] \]

\[ = \neg[\exists x_1 (\text{man}_x) \land \neg[\exists x_2 \exists y_2 \exists y_1 [\text{loves}_x(y_2) \land \text{woman}_y \land x_1 = y_2 \land y_1 = y_2]]] \]

\[ = \forall x_1 [\text{man}_x \rightarrow \exists y_1 [\text{loves}_x(y_1) \land \text{woman}_y]] \]

(3) \( \overline{\text{man}_x(y) \text{loves}_y(y) \text{woman}_y} \)

\[ \exists y_1 [\text{woman}_y \land \forall x_1 [\text{man}_x \rightarrow \text{loves}_x(y_1)]] \]

In EG, every must have a scroll structure and its scope is always confined to this cut; while existential quantifier a/some can get free scope by interpreting it at different positions with respect to cuts. It is natural that the scope of a line of identity is the area where it occurs, confined within its immediately enclosing cut if there is one. Further assume that a third-person singular pronoun in Existential Graphs introduces a line of identity with one end unattached (loose end), which needs to be linked to another fully-attached line of identity for interpretation.

(4) **Line linking condition:** A quantifier line can link to a pronoun line if the pronoun is encountered later than the quantifier line.

Like DRS, Existential Graphs can be considered as mental representation of speakers and hearers, with each subgraph representing a piece of information, packaged into a whole by conjunction and/or enclosure in a certain universe of discourse represented by the sheet of assertion. These subgraphs constitute the explicit information in EG. On the other hand, Existential Graphs can also contain implicit information inferred from the discourse. This implies that graphs (information) may be added or copied into the sheet of assertion, a kind of context change. Peirce devised five rules of inference for importing and exporting information into and out of the contexts and stated the conditions for such operations:

**Rule 1 Insertion:** In a negative context, any graph or subgraph may be inserted.
Two lines of identity (or portions of lines) oddly enclosed on the same area may be joined. This corresponds to weakening a consequent.

**Rule 2 Erasure:** In a positive context, any graph or subgraph or portion of a line of identity may be erased.

**Rule 3 Iteration:** If a graph or subgraph $P$ occurs in a context $\xi$, another copy of $P$ may be written in the same context or in any context nested in $\xi$.

**Rule 4 Deiteration:** Any graph that could have been derived by iteration may be erased: any graph or subgraph $p$ may be erased if a copy of $p$ occurs in the same context or any containing context.

**Rule 5 Double cut:** Two cuts with nothing between them may be erased or inserted around any graph or collection of graphs in any context.

3. Uniqueness in intersentential anaphora

(5) A man walks in the park. He whistles.

The starting graph for this discourse is: $\text{man} \rightarrow \text{walks in the park. } \rightarrow \text{whistles}$, in which he second line for the pronoun does not get a proper interpretation. An analysis would produce Graph 1 for this sentence, in which the line standing for $\text{man}$ branches to link with the loose end of the line standing for the pronoun $\text{he}$. It is interpreted as there is an $x$ such that $x$ is man and $x$ walks in the park and $x$ whistles (a Geachian analysis). This graph does not capture the uniqueness implication of the pronoun. It is the graph for a man walks in the park and whistles or a man who walks in the park whistles, etc.

$$\exists x_1[\text{man } x_1] \land \exists x_2[\text{walks-in-the-park } x_2] \land \exists x_3[\text{whistles } x_3] \land x_1=x_2=x_3$$

$$=\exists x_1[\text{man } x_1 \land \text{walks-in-the-park } x_1 \land \text{whistles } x_1]$$

Though there has been much debate on whether donkey pronouns display uniqueness implication, it is relatively clear that uniqueness is most obviously manifested in donkey anaphora involving conjunctive discourse (Evans 1977, Cooper 1979, Kadmon 1990, Dekker 2001, among many others).

In order for linking possible, one line should nest another. Peirce’s inference rules make this possible: (1) double-negate part of the second conjunct without change in truth value, linearly $((\neg\neg))$—whistles. (2) Iterate the subgraph $\text{man} \rightarrow \text{walks in the park}$ into the area between two cuts, we have $\text{man} \rightarrow \text{walks in the park}$ (3)$\text{whistles}$. (3) Link the line representing $\text{man}$ in the sheet of assertion to the line representing the pronoun attached to $\text{whistles}$, because the scope of the line representing $\text{man}$ nests the line representing $\text{he}$. As a result, we obtain a rather complicated graph (Graph 2) for a simple conjunction: A man walks in the park. He whistles. In this way, the pronoun is got bound indirectly by the existential quantifier.
This well-formed graph can be considered as the logical form that feeds semantic interpretation. Interpreting the graph in an inward-going manner, we have following logical formula of first-order logic.

\[
(7) \exists x (\text{man } x \land \text{walks in the park } x) \land \exists y (\text{whistles } y \land \neg \exists z (x = z \land z = y))
\]

The underlined subformula captures the Russellian uniqueness of the pronoun, which is treated a variable. The derivation is compositional in that the formulae for ‘a man walks in the park’ and ‘he whistles’ are proper subparts of the whole. Therefore we see that uniqueness is a by-product of linking (or binding). Pronouns themselves do not contain this meaning in semantics, which is derived by the requirement that pronouns, being variables, should be linked or bound.

4. Non-uniqueness and uniqueness in donkey anaphora

Though conjunction always has uniqueness implication, donkey anaphora may or may not exhibit uniqueness implication, depending mainly on contexts.

(8) a. If a man has a daughter, he thinks she is the most beautiful girl in the world.
   b. If a bishop meets another bishop, he blesses him.

First consider non-uniqueness (to simplify, the following classical sentence is used):

(9) a. If a farmer owns a donkey, he beats it.
   b. (farmer—owns—donkey (——beats——))

In (9b), there are four lines of identity, representing farmer, donkey, he, and it. The two lines occurring within the intermediate area are properly linked to farmer and donkey and to a relation of owning; but the two lines representing the two pronouns occurring within the innermost area are only linked to a relation of beating, their loose ends are not properly linked, so they need to be instantiated; otherwise it is interpreted as something beats something, which is not the reading we get. One way for a pronoun to be instantiated in EG is to link its loose end to another line that is fully interpreted. In the linear notation (9b), there are two fully interpreted lines, the lines of a farmer and a donkey, scoping over the innermost cut, the two existential quantifiers are able to link/bind the pronouns within the innermost cut: \( x_1 = x_2 = z_3 = z_4, y_1 = y_2 = z_4 = z_2 \), as diagrammed in Graph 3 (Sowa 2000). The entire graph is rendered into the following classical predicate logic language:

\[
(10) \neg [(\exists x (\text{farmer } x_1) \land \exists z_1 (\text{owns } z_1 z_2) \land \exists y_1 (\text{donkey } y_1) \land x_1 = z_1 \land y_1 = z_2) \land
\neg (\exists z_3 (\text{beats } z_3 z_4) \land \exists x_2 (x_2 = z_3 \land x_1 = x_2) \land \exists y_2 (y_2 = z_4 \land y_1 = y_2))]
\]
Therefore, donkey anaphora turns out to be typical case of variable binding (Barker & Shan 2008). Being typical case of variable binding does not exclude uniqueness in certain contexts. Next let’s see how uniqueness implication is derived: Iterate the subgraph —owns—donkey into the inner area of scroll, which makes the indefinite —donkey in the same area with the pronoun.

(11)a. \( \text{farmer} \rightarrow \text{owns} \rightarrow \text{donkey} \rightarrow \text{beats} \)

b. \( \text{farmer} \rightarrow \text{owns} \rightarrow \text{donkey} \rightarrow \text{owns} \rightarrow \text{donkey} \rightarrow \text{beats} \)

Repeat the operation in Graph (2). As a result, Graph 4 can be translated into the final (12), which carries a relative uniqueness implication too.

(12) \( \forall x (\text{farmer} x \rightarrow \exists y (\text{donkey} y \rightarrow \text{owns} \rightarrow \text{donkey} y \rightarrow \text{beats} \rightarrow \text{owns} \rightarrow \text{donkey} y \rightarrow \text{beats} \rightarrow \text{owns} \rightarrow \text{donkey} y \rightarrow \text{beats}) \)

5. Motivating the Line linking condition and restricting the inference rule of Double Cut

6. Summary

Peirce’s logic may be useful to the study of donkey anaphora as already pointed out by many scholars (Pagin and Westerståhl 1993, van den Berg 1995, Sowa 2000, Johnson-Laird 2002, Pietarinen 2006, Asher 2007, Rellstab 2008). Anyway, the assumptions used in this system are quite standard, without radical departures from classical semantics: there is nothing peculiar with the indefinite (DRT), it is just an existential quantifier; there is nothing peculiar with the pronoun (E-type), it is just a variable; and there is nothing peculiar with conjunction (DPL), it is just a truth conditional operator. Then something must be peculiar. In Existential Graphs, the peculiar thing is the importing and exporting of information into and out of the contexts, in the form of five inference rules, which are standard natural deduction system.