

Optimal investment strategies and hedging of derivatives in the presence of transaction costs

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References

- E. Aurell and P. Muratore-Ginanneschi, “Financial Friction and Multiplicative Markov Market Game”, *International J. of Theoretical and Applied Finance (IJTAF)* 3, No. 3 (2000) 501-510

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- E. Aurell and P. Muratore-Ginanneschi, “ Optimal hedging of derivatives with transaction costs”, *in prep.*

Market model with transaction costs

Stock and bond model

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- Time $t + dt$: $W_{t+1}^{\text{Stocks}} = [u_t \rho_t + \Delta \chi_t] W_t$

$$W_{t+1} = [1 + \rho_t(u_t - 1) - \Delta F_\gamma(\chi_t)] W_t$$

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- χ_t : Control strategy
- ΔF_γ : Transaction costs

Optimal control problem

- Multiplicative Markov model

$$\Delta\rho_t = \frac{\rho_t + (u_t - 1)\rho_t + \Delta\chi_t}{1 + \rho_t(u_t - 1) - \Delta F_\gamma(\chi_t)} - \rho_t$$

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- A natural choice of the utility is

$$\lambda(x, t|T) = \mathbb{E}_{\rho_t=x} \left\{ \ln \frac{W_T}{W_t} \right\}$$

Continuum time description

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- The dynamics is modelled by the SDE's

$$\begin{aligned} dW_t &= [\mu \rho_t - \gamma \mathcal{F}(f)] W_t dt + \sigma \rho_t W_t dB_t \\ d\rho_t &= [f + a + \gamma \rho_t \mathcal{F}(f)] dt + b dB_t \end{aligned}$$

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$$a = \rho_t(1 - \rho_t)(\mu - \sigma^2 \rho_t)$$

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$$b = \sigma \rho_t(1 - \rho_t)$$

Oseledec's theorem

The stochastic control is determined as a function of ρ_t and t , by maximizing the expectation value of the wealth growth:

$$\lambda(x, t; T) = \mathbb{E}_{\rho_t=x} \left\{ \int_t^T ds [V(\rho_s) - \gamma \mathcal{F}(\rho_s)] \right\}$$

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Oseledec's theorem predicts

$$\lim_{T-t \uparrow \infty} \lambda(x, t|T) = \lim_{T-t \uparrow \infty} (T - t) \ell$$

Hamilton-Bellman-Jacobi formalism

Dynamic programming equation

$$\partial_t \lambda + [f + a + \gamma x \mathcal{F}] \partial_x \lambda + \frac{b^2}{2} \partial_x^2 \lambda + V - \gamma \mathcal{F} = 0$$

$$\lambda(x, T; T) = 0$$

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If borrowing and short-selling is not allowed, the probability measure of ρ is conserved in the interval $[0, 1]$

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Dimensional analysis

$$[\lambda] = [x] = 0, \quad [\sigma^2] = [\mu] = [f] = [1/t], \quad [\gamma \mathcal{F}] = [1/t]$$

Optimal control

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$$\partial_t \lambda + a \partial_\rho \lambda + \frac{b^2}{2} \partial_\rho^2 \lambda + V = 0$$

$$\partial_\rho \lambda \Big|_{\rho=\rho_{Max}} = -\frac{\gamma}{1 - \gamma \rho_{Max}}, \quad \partial_\rho \lambda \Big|_{\rho=\rho_{min}} = \frac{\gamma}{1 + \gamma \rho_{min}}$$

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- “Quadratic” transaction costs $\mathcal{F} = |f|^2$

$$\partial_t \lambda + a \partial_\rho \lambda + \frac{(\partial_\rho \lambda)^2}{4\gamma(1 - \rho \partial_\rho \lambda)} + \frac{b^2}{2} \partial_\rho^2 \lambda + \mu \rho - \frac{\sigma^2 \rho^2}{2} = 0$$

Investment strategies

In the **absence** of transaction costs market fall-outs can be balanced without constraints

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- Dimensionless order parameter

$$\varepsilon = \begin{cases} \gamma & \text{linear transaction costs} \\ \sigma^2 \gamma & \text{quadratic transaction costs} \end{cases}$$

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- $\lambda = (T - t) \frac{\mu^2}{2\sigma^2} + \varphi(T - t, x, \varepsilon), \quad x = \rho - \rho^*$

Scaling analysis

- Every quantity is rescaled by a power of ε , e.g.

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$$x \Rightarrow \varepsilon^{\omega_x} x_\varepsilon$$

- Controls are used to match diffusion and utility.
- Example: quadratic friction

$$\begin{aligned} O(\partial_t \lambda) &= O\left(\frac{(\partial_x \lambda)^2}{4\gamma(1-x\partial_x \lambda)}\right) &\Rightarrow \omega_\varphi - \omega_t &= 2(\omega_\varphi - \omega_x) - 1 \\ O(\partial_x^2 \lambda) &= O\left(\frac{(\partial_x \lambda)^2}{4\gamma(1-x\partial_x \lambda)}\right) &\Rightarrow \omega_\varphi - 2\omega_x &= 2(\omega_\varphi - \omega_x) - 1 \\ O\left(\frac{\sigma^2 x^2}{2}\right) &= O\left(\frac{(\partial_x \lambda)^2}{4\gamma(1-x\partial_x \lambda)}\right) &\Rightarrow 2\omega_x &= 2(\omega_\varphi - \omega_x) - 1 \end{aligned}$$

Normal forms

- Linear friction

$$\lambda(x, T - t, \varepsilon) = (T - t) \frac{\mu^2}{2\sigma^2} + \varepsilon^{4/3} \varphi\left(\frac{T - t}{\varepsilon^{2/3}}, \frac{x}{\varepsilon^{1/3}}; 1\right) + \text{h.o.t}$$

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- Quadratic friction

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$$\partial_t \lambda + \frac{(\partial_x \lambda)^2}{4\gamma} + \frac{D^2}{2} \partial_x^2 \lambda + \frac{\mu^2}{2\sigma^2} - \frac{\sigma^2 x^2}{2} = 0$$

Properties of the solutions

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“Linear t.c.”:

$$\ell = \frac{\mu^2}{2\sigma^2} - \frac{(9 D^2 \sigma^2 \epsilon^2)^{1/3}}{2}$$
$$L^* \propto \epsilon^{1/3}$$

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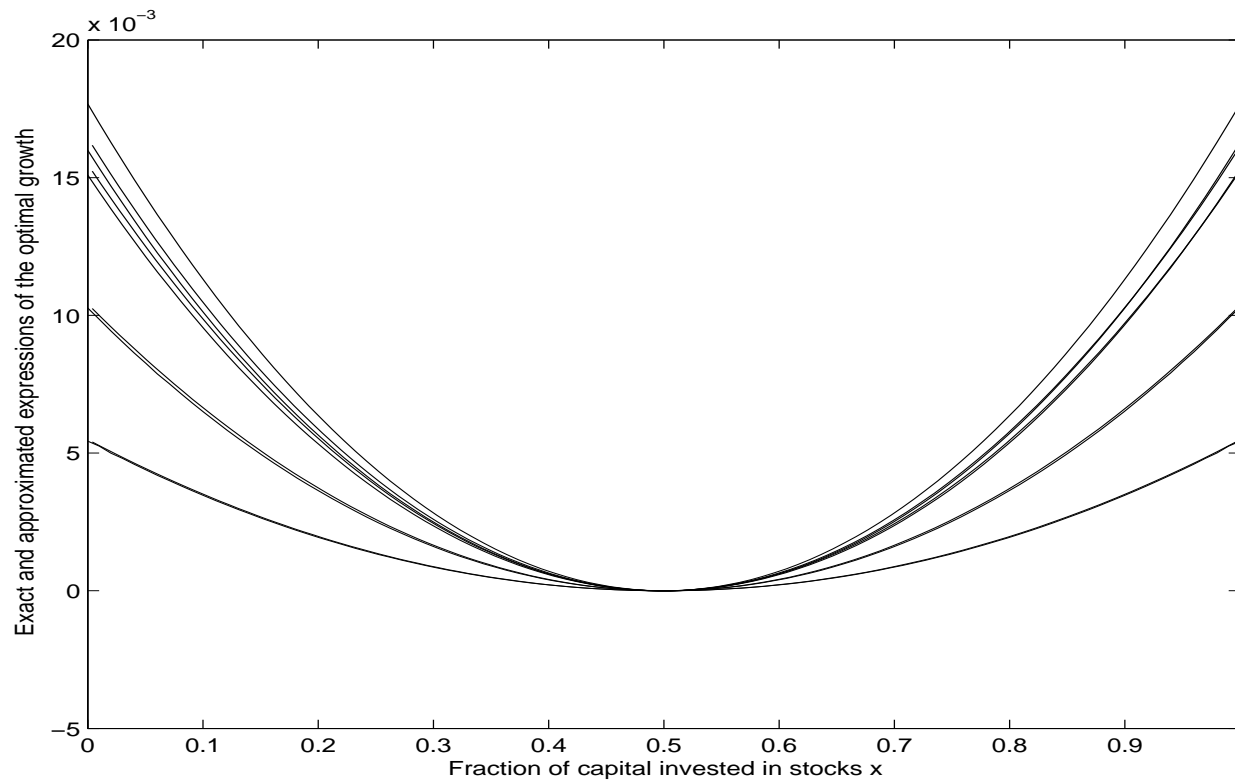
Quadratic friction:

$$\ell = \frac{\mu^2}{2\sigma^2} - D^2 \sqrt{\frac{\epsilon}{2}}$$
$$L^* \propto \epsilon^{1/4}$$

Control potential for quadratic friction

$$f(x, T - t) = -\partial_x \lambda(x, T - t)$$

for $\mu = \sigma^2/2$, $\sigma = 10^{-2}$ and $\gamma = 10^2$.



Inclusion of a stock derivative

Dynamical equations

$$dW = W [(\mu\rho + \mu_d\eta) dt + (\sigma\rho + \sigma_d\eta) dB_t - (\gamma\mathcal{F} + \gamma_d\mathcal{F}_d) dt]$$

$$d\rho = [f + a + \rho(\gamma\mathcal{F} + \gamma_d\mathcal{F}_d)] dt + b dB_t$$

$$d\eta = [f_d + a_d + \eta(\gamma\mathcal{F} + \gamma_d\mathcal{F}_d)] dt + b_d dB_t$$

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Derivative price

$$\begin{aligned} \frac{dC}{C} &= \frac{1}{C} \left[\partial_t C + \mu p \partial_p C + \frac{\sigma^2 p^2}{2} \partial_p^2 C \right] dt + \frac{\sigma p \partial_p C}{C} dB_t \\ &:= \mu_d dt + \sigma_d dB_t \end{aligned}$$

Non convex utility

- In the absence of transaction costs

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- Introducing coordinates along the marginal and stable subspaces of the Hessian gives

$$V = \mu \zeta + (\sigma \mu_d - \mu \sigma_d) \frac{\sigma_d \zeta - \sigma \vartheta}{\sigma^2 + \sigma_d^2} - \frac{\sigma^2 \zeta^2}{2}$$

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- Solvability condition: Black and Scholes

$$\mu_d = \frac{\mu}{\sigma} \sigma_d \Rightarrow \partial_t C + \frac{\sigma^2 p^2}{2} \partial_p^2 C$$

Summary

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- Black and Scholes provide a solvability condition
- The hedging is defined by the solution of an Hamilton-Jacobi-equations