Small scale statistics in viscoelastic turbulent flows

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Outline

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Polymers

number of monomers N

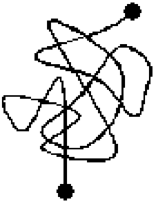
monomer length a

radius of gyration $r_0 = N^{3/5} a$

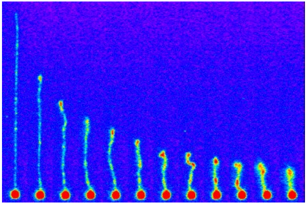
typically...

$N \sim 10^6 \div 10^7, \; r_0 \sim 0.1 \mu m$

coiled state



stretched state



$r \ll r_{\max} \;\Rightarrow\;$ linear relaxation

[from experiments with DNA molecules]

relaxation time

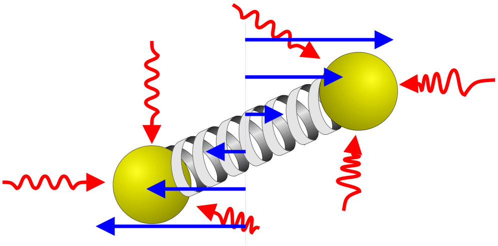
$$\tau = \frac{\mu r_0^3}{K_B T}$$

[Perkins et al., Science **264**, 819 (1994)]

Outline

Dumbbell model

- Flow
  stretching
- Brownian motion
  thermal fluctuations
- Spring
  entropic tendency to coil up



$$\dot{\mathbf{r}} = (\nabla \mathbf{u})^T \boldsymbol{r} - \frac{1}{\tau}\mathbf{r} + (2r_0^2/\tau)^{1/2}\xi$$

Outline

Oldroyd-B model (I)

Single polymer dynamics → Hydrodynamical description

1. $\sigma_{ij} \equiv r_0^{-2} < r_i r_j >$ conformation tensor
2. $\frac{Du_i}{Dt} = f_i + \frac{1}{\rho}\frac{\partial T_{ij}}{\partial x_j}$; $\boldsymbol{T} = \boldsymbol{T}^N + \boldsymbol{T}^P$ stress tensor
   - $T_{ij}^N = -p\delta_{ij} + \mu[(\nabla_j u_i + \nabla_i u_j) - \frac{2}{3}\nabla_k u_k \delta_{ij}]$
   - $T_{ij}^P = nK_0 r_0^2 \sigma_{ij}$

where

n is polymer concentration

$$\eta = \frac{nK_0 r_0^2 \tau}{\mu} \; ; \quad \mu = \nu\rho$$

Oldroyd-B model (II)

For an incompressible ($\nabla \cdot \boldsymbol{u} = 0$) velocity field we obtain:

$$\begin{cases} \partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \nu \Delta \boldsymbol{u} + \boldsymbol{F} + \underbrace{\frac{\eta\nu}{\tau} \nabla (\boldsymbol{\sigma} - \mathbf{1})}_{feedback} \\ \partial_t \boldsymbol{\sigma} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\sigma} = \underbrace{(\nabla \boldsymbol{u})^T \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot (\nabla \boldsymbol{u})}_{stretching} - \underbrace{\frac{1}{\tau} (\boldsymbol{\sigma} - \mathbf{1})}_{relaxation} \end{cases}$$

Outline

1. Viscoelastic fluids
   - Polymers
   - Dumbbell model
   - Oldroyd-B model
   - Uniaxial model
   - Lumley criterion
2. Small scale observables
   - Accelerations
   - Viscous dissipation
3. Conclusions and future work

Uniaxial model (I)

Hypothesis

[Fouxon et al., Phys. Fluids **15**, 7, 2060 (2003)]

1. polymers are stretched: $|\boldsymbol{r}| \gg r_0$
2. $\boldsymbol{\sigma} = \boldsymbol{B} \otimes \boldsymbol{B}$

$$\boldsymbol{\sigma}\, \boldsymbol{v}^{(\alpha)} = \lambda_\alpha \boldsymbol{v}^{(\alpha)}\,; \quad \lambda_1 \gg \lambda_2 \gg \ldots \gg \lambda_d$$

$$\Rightarrow \sigma_{ij} = \sum_{\alpha=1}^{d} \lambda_\alpha v_i^{(\alpha)} v_j^{(\alpha)} \simeq \lambda_1 v_i^{(1)} v_j^{(1)}$$

$$B_i = \sqrt{\lambda_1}\, v_i^{(1)} \quad \Rightarrow \quad \boxed{\sigma_{ij} = B_i B_j}$$

Uniaxial model (II)

$$|\boldsymbol{r}| \gg r_0 \quad \Rightarrow \quad \dot{\mathbf{r}} = (\nabla \mathbf{u})^T \boldsymbol{r} - \frac{1}{\tau}\mathbf{r} + \sout{(2r_0^2/\tau)^{1/2}}\boldsymbol{\xi}$$

$$[\boldsymbol{\sigma} - \mathbf{1} \;\rightarrow\; \boldsymbol{\sigma} \;\; \text{in Oldroyd-B eq.}]$$

In this limit we get

$$\begin{cases} \partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} &= -\nabla p + \nu\Delta\boldsymbol{u} + \boldsymbol{F} + \frac{\eta\nu}{\tau}(\boldsymbol{r}\cdot\nabla)\boldsymbol{r} \\ \partial_t \boldsymbol{r} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{r} &= (\nabla\boldsymbol{u})^T\cdot\boldsymbol{r} - \frac{1}{\tau}\boldsymbol{r} \end{cases}$$

Finally, with the rescaling $\boldsymbol{B} = (\eta\nu/\tau)^{1/2}\boldsymbol{r}$

$$\begin{cases} \partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} &= -\nabla p + \nu\Delta\boldsymbol{u} + \boldsymbol{F} + (\boldsymbol{B}\cdot\nabla)\boldsymbol{B} \\ \partial_t \boldsymbol{B} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{B} &= (\nabla\boldsymbol{u})^T\cdot\boldsymbol{B} - \frac{1}{\tau}\boldsymbol{B} \end{cases}$$

Outline

Lumley criterion

Coil-stretch transition: $\dot{\mathbf{B}} = (\nabla \mathbf{u})^T \boldsymbol{B} - \frac{1}{\tau}\mathbf{B}$

$$Wi \equiv \lambda\tau \quad \Rightarrow \quad \begin{cases} Wi \ll 1 & \text{polymers are coiled} \\ Wi \gg 1 & \text{polymers are stretched} \end{cases}$$

with λ the Lyapunov exponent

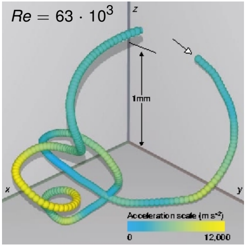
$$\tau(\ell) \sim \epsilon^{-1/3}\ell^{2/3} \quad \text{(K41)}$$

Lumley criterion: $\tau(\ell) \sim \tau \quad \Rightarrow$ polymers are active at scales $\ell < \ell_L$ where...

$$\ell_L : 1 \sim Wi(\ell_L) = \frac{\tau}{\tau(\ell_L)} \quad \Rightarrow \quad \boxed{\ell_L \sim (\epsilon\tau^3)^{1/2}}$$

Outline

Newtonian accelerations



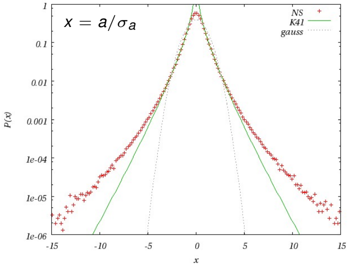
[La Porta et al., Nature **409**, 1017 (2001)]

$$a \equiv \lim_{\tau \to 0} \frac{\delta u(\tau)}{\tau} \sim \frac{\delta u(\tau_\eta)}{\tau_\eta} \quad \Rightarrow \quad a \sim \frac{\delta u(\eta)^2}{\eta}$$

$$\delta u(\ell) = u_0 \left(\frac{\ell}{L}\right)^h \quad \Rightarrow \quad a \sim \frac{u_0^2}{L} Re^{-\frac{2h-1}{1+h}}$$

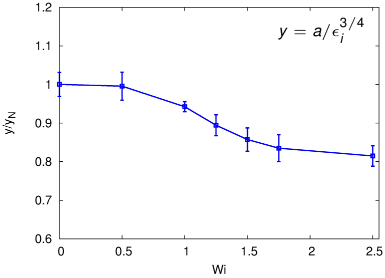
K41 $\Rightarrow h = 1/3 \quad \Rightarrow \quad a^2 \sim Re^{1/2}$

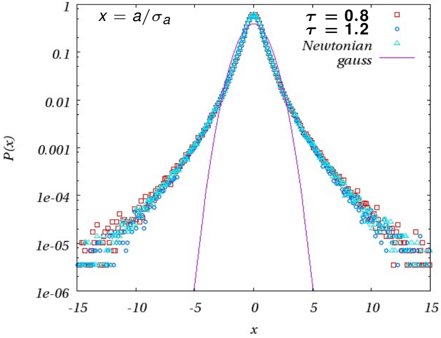
$$\Rightarrow \quad a \sim \epsilon^{3/4} \nu^{-1/4}$$



Viscoelastic accelerations (I)

$$a \equiv \frac{d}{dt}\boldsymbol{u} = -\nabla p + \nu \Delta \boldsymbol{u} + \boldsymbol{F} + (\boldsymbol{B} \cdot \nabla)\boldsymbol{B}$$





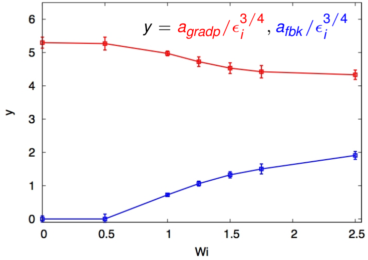
$\nu = 6 \cdot 10^{-3}$

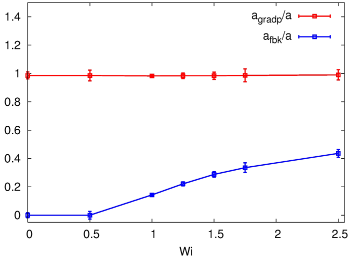
$Re \simeq 504$, $R_\lambda \simeq 87$

$\tau_L \simeq 2.81$, $\tau_\eta \simeq 0.08$

$\lambda_N \tau_\eta \sim 0.1 \rightarrow Wi = \lambda_N \tau$

Viscoelastic accelerations: contributions (II)





τ	τ/τ_η	Wi	σ_{gradp}	σ_{dissi}	$\sigma_{forcing}$	σ_{fbk}	σ_a
Newton			4.726	0.706	0.296		4.8
0.4	5.0	0.50	4.730	0.707	0.298	$< 10^{-30}$	4.8
0.8	10.0	1.00	4.329	0.643	0.297	0.631	4.4
1.0	12.5	1.25	4.015	0.655	0.293	0.902	4.1
1.2	15.0	1.50	3.829	0.687	0.289	1.120	3.9
1.4	17.5	1.75	3.652	0.702	0.300	1.243	3.7
2.0	25.0	2.50	3.533	0.774	0.274	1.557	3.6

Viscoelastic accelerations: contributions (III)

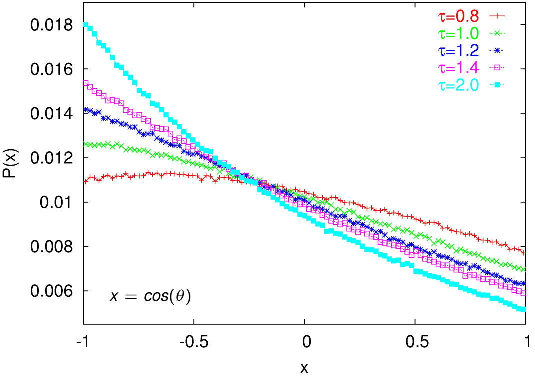
leading contributions:

pressure gradient $-\nabla p \rightarrow \boldsymbol{a}_{gradp}$

feedback $(\boldsymbol{B} \cdot \nabla)\boldsymbol{B} \rightarrow \boldsymbol{a}_{fbk}$

$$cos(\theta) = \frac{\boldsymbol{a}_{fbk} \cdot \boldsymbol{a}_{gradp}}{|\boldsymbol{a}_{fbk}||\boldsymbol{a}_{gradp}|}$$

Orientation pdf



Polymers tend to oppose to pressure gradients

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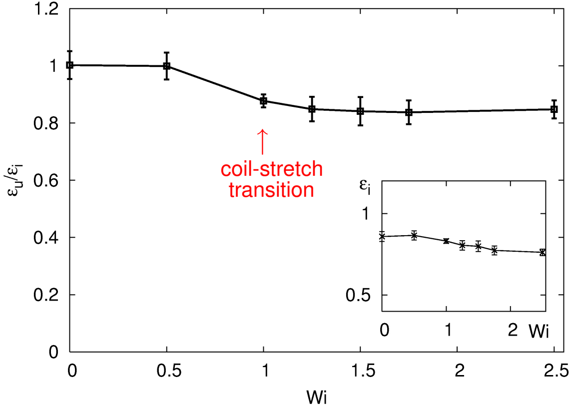
Viscous dissipation (I)

$$\epsilon_u = \frac{\nu}{2} < \sum_{ij} (\partial_i u_j + \partial_j u_i)^2 >; \quad \epsilon_i =< \boldsymbol{u} \cdot \boldsymbol{F} >$$



$\nu = 6 \cdot 10^{-3}$

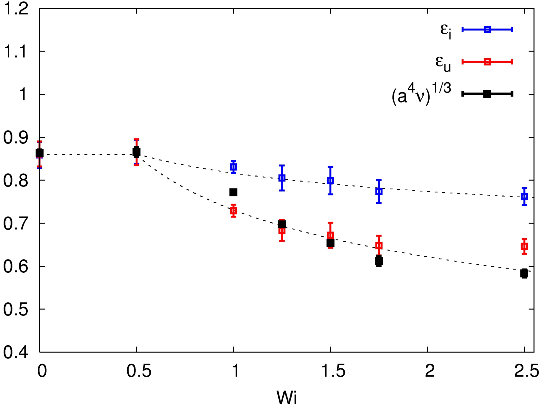
$R_\lambda \simeq 87$; $Re \simeq 504$

$\tau_L \simeq 2.81$; $\tau_\eta \simeq 0.08$

$$\lambda_N \tau_\eta \sim 0.1 \rightarrow Wi = \lambda_N \tau$$

Viscous dissipation (II)

$$\epsilon_u = \frac{\nu}{2} < \sum_{ij} (\partial_i u_j + \partial_j u_i)^2 >; \quad \epsilon_{el} =< \frac{|\boldsymbol{B}|^2}{\tau} >; \quad \epsilon_i =< \boldsymbol{u} \cdot \boldsymbol{F} >$$



Newtonian flow

$a \sim \epsilon^{3/4} \nu^{-1/4}$

$\epsilon_i = \epsilon_u = \epsilon$

Viscoelastic flow

$a \sim \epsilon_u^{3/4} \nu^{-1/4}$

$\epsilon_i = \epsilon_u + \epsilon_{el}$

Conclusions...

1. Polymers reduce rms accelerations, but they substantially do not affect the shape of their probability distributions.
2. Viscous dissipation is reduced by the addition of polymers. At moderate values of elasticity, ϵ_u/ϵ_i displays a very weak dependence on *Wi*.
3. The first two points suggest the idea that turbulence is still active below the Lumley scale ℓ_L, though with a reduced energy flux.

...and future work

1. Analysis with a more realistic (FENE-P) model
2. Comparison with experimental results