Elastic Turbulence in 2D Viscoelastic Flows

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1 Introduction

It is well known that the addition of small amounts of long chain polymers produces dramatic effects on flowing fluids. A remarkable effect, recently observed experimentally, is the onset of "elastic turbulence" in the limit of very low Reynolds number *Re*, provided elasticity is high enough [1]. In this regime, the polymer solution displays features typical of turbulent flows. Consequently, elastic turbulence has been proposed as an efficient technique for mixing in very low *Re* flows as, e. g., microchannel flows [2]. Despite its wide technological interest, elastic turbulence is still poorly understood from a theoretical point of view. Recent predictions are based on simplified versions of viscoelastic models and on the analogy with MHD equations [3].

In this study we investigate the phenomenology of elastic turbulence by means of direct numerical simulations (DNS). The remarkable agreement with experimental results suggests the possibility to understand elastic turbulence on the basis of known viscoelastic models.

2 Viscoelastic Model

The dynamics of the dilute polymer solution is described by the linear viscoelastic Oldroyd-B model:

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \frac{2\eta\,\nu}{\tau}\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \boldsymbol{f} \tag{1}$$

$$\partial_t \boldsymbol{\sigma} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{\sigma} = (\boldsymbol{\nabla}\boldsymbol{u})^T \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot (\boldsymbol{\nabla}\boldsymbol{u}) - \frac{2(\boldsymbol{\sigma} - \boldsymbol{1})}{\tau} \tag{2}$$

where $\boldsymbol{u}$ is an incompressible, two-dimensional, velocity field; the matrix of velocity gradients is defined as $(\boldsymbol{\nabla u})_{ij} = \partial_i u_j$ and $\boldsymbol{f}$ is an external forcing; the symmetric positive definite matrix $\boldsymbol{\sigma}$ is the conformation tensor of polymer molecules and $\mathbf{1}$ the unit tensor. The (slowest) polymer relaxation time is denoted by τ; ν is the kinematic viscosity of the solvent and η is the zero-shear contribution of polymers to the total viscosity $\nu_T = \nu(1+\eta)$ of the solution.

3 Numerical results

The equations of motion are integrated, for a Kolmogorov flow configuration corresponding to $\boldsymbol{f} = (F\cos(y/L), 0)$, by means of a pseudo-spectral method implemented on a grid of resolution 512^2 with periodic boundary conditions; the Weissenberg number Wi, controlling elastic instabilities, is varied in a wide range and Re is kept fixed at a small value.

It is known that DNS of viscoelastic models are limited by instabilities associated with the loss of positiveness of the conformation tensor [4], which are particularly important at high elasticity and restrict the possibility to investigate the elastic turbulent regime by direct integration of Eqs. (1)-(2). To overcome this problem we developed an algorithm based on a Cholesky decomposition of the conformation matrix ensuring symmetry and positive definiteness [5]. To further control numerical instabilities, the simulations have been performed adding a small diffusivity κ for polymers to Eq. (2), the corresponding Schmidt number $Sc \equiv \nu/\kappa$ being always greater than 5×10^2.

The polymer solution flow is found to display features of a strongly non-linear state such as irregular temporal behaviour and spatial disorder (see Fig. 1).

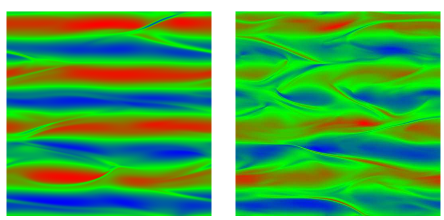

Fig. 1. Vorticity field from DNS, for Wi slightly above the elastic instability threshold (left panel) and for a larger value of Wi (right panel).

At moderate values of Wi, a secondary flow develops in the form of thin filaments superimposed to the basic flow. These small-scale structures are elastic waves reminiscent of Alfvén waves propagating in a plasma. The possibility to observe elastic waves in polymer solutions was theoretically predicted within a simplified uniaxial model [3]. At higher values of elasticity the flow develops active modes at all the scales and finally reaches a turbulent-like state characterized by a power law spectrum of velocity fluctuations slightly steeper than k^{-3}, in excellent agreement with laboratory observations [1].

Since in this elastic turbulence regime the flow is smooth, a suitable characterization of mixing is given by the Lagrangian Lyapunov exponent λ_L. The behaviour of λ_L as a function of Wi is shown in Fig. 2; in the inset of this figure we also plot the Cramer function $G(\gamma)$ which is defined from the probability distribution of Lyapunov exponent fluctuations $P_t(\gamma) \sim \exp(-tG(\gamma))$.

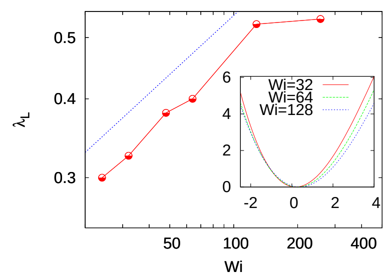

Fig. 2. Lagrangian Lyapunov exponent at varying Weissenberg number ($Re = 1$); the straight dashed line has slope 0.31. Inset: Cramer function $G(\gamma)$.

We observe a growth approximately following a power law $\lambda_L \sim Wi^{0.3}$, accompanied by larger and larger fluctuations γ. In particular, the distribution of γ becomes asymmetric with a larger relative probability of positive fluctuations, in close analogy to the usual behaviour of Newtonian fluids at growing Re.

4 Conclusions

In conclusion, we numerically reproduced the basic phenomenology of elastic turbulence in a 2D configuration. It would be very interesting to extend our work to a 3D setup, in order to better compare with experiments and gain more insight on the basic physical mechanisms underlying the phenomenon.

References

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