Bayesian analysis of space shuttle Challenger data

The space shuttle Challenger and its crew of seven people was destroyed soon after its launch on Jan 28, 1986. The disaster was due to leakage of gas from one of the fuel tanks. This was caused by damage to one of the six field-joint O-rings, which are insulating rings made of rubber. At the time of the disaster the temperature was 31 degrees Fahrenheit, which was unusually low. There is data available on the number of field-joint O-rings showing signs of damage and the launch temperature on 23 previous flights.

We analyze the Challenger data using a logistic regression model, which assumes that conditionally on the parameters, the number of damaged O-rings is binomially distributed with probability which depends on the launch temperature. That is,

\[
[Y_i | \beta_0, \beta_1] \sim \text{Bin}(6, \pi_i), \quad \logit(\pi_i) = \beta_0 + \beta_1 x_i, \quad i = 1, \ldots, n,
\]

where \(n = 23\) and \(y_i\) is the number of damaged O-rings and \(x_i\) is the launch temperature on the \(i\)th flight. Let \(Y^*\) be the number of damaged O-rings at the launch temperature \(x^* = 31\). We assume that \(Y = (Y_1, \ldots, Y_n)\) and \(Y^*\) are conditionally independent given \((\beta_0, \beta_1)\) and that \(p(y^* | \beta_0, \beta_1)\) is the binomial distribution with sample size 6 and “success” probability \(\logit^{-1}(\beta_0 + \beta_1 x^*)\).

Since formulating the prior directly for the coefficients \((\beta_0, \beta_1)\) is difficult, we instead formulate a prior for the parameters

\[
\alpha_1 = \logit^{-1}(\beta_0 + \beta_1 60), \quad \alpha_2 = \logit^{-1}(\beta_0 + \beta_1 80).
\]

We take \(\alpha_1\) and \(\alpha_2\) to be independent and uniform \(\text{Uni}(0, 1)\). Since \((\beta_0, \beta_1)\) can be expressed uniquely in terms of \((\alpha_1, \alpha_2)\), this implies a prior also for \((\beta_0, \beta_1)\). One of the tasks is to try to understand what this implied prior looks like.

- Solve \((\beta_0, \beta_1)\) in terms of \((\alpha_1, \alpha_2)\).
- Produce a contour plot of the prior in \((\beta_0, \beta_1)\)-plane, and add points simulated from the prior on top of the contour plot.
- Write a MCMC algorithm (e.g., independent Metropolis–Hastings with proposals coming from the prior; or random-walk Metropolis; or put the whole model into BUGS and let it select the sampler for you). Your
are free to select the details of the sampler (such as the parametrization on which operates) yourself, but you must then be able to explain your choices. Draw a contour plot of the posterior in $(\beta_0, \beta_1)$-plane, and add points simulated from the posterior on top of the contour plot.

- Plot the predictive distribution $y^* \mapsto p(y^* \mid y)$, where $y^* = 0, 1, \ldots, 6$. 