1. Let $H$ be a separable Hilbert space and let $(e_i)_{i=1}^{\infty}$ be an orthonormal basis in $H$. Prove the generalized Parseval formula

$$\langle h, h' \rangle_H = \sum_{i=1}^{\infty} \langle h, e_i \rangle_H \langle h', e_i \rangle_H.$$ 

2. Let $H$ be a separable Hilbert space and let $(e_i)_{i=1}^{\infty}$ be an orthonormal basis in $H$. Show that the representation

$$h = \sum_{i=1}^{\infty} \langle h, e_i \rangle_H e_i$$

is unique when the basis is fixed.

3. Let $W = (W(h) : h \in H)$ be an isonormal Gaussian process. Show that the mapping $h \mapsto W(h)$ is linear.

4. Let $M = (M_t)_{t \in [0,1]}$ be a centered Gaussian martingale, i.e. a centered Gaussian process with covariance function $K(t,s) = f(\min(t,s))$. What is the natural Hilbert space $H$ associated with $M$?

5. Let $M = (M_t)_{t \in [0,1]}$ be a centered Gaussian process with covariance function $K(t,s) = f(\min(t,s))$. What is the natural Hilbert space $H$ associated with $M$?