1. Let \( u, v \in \mathcal{E}_a \). Show the Itô isometry

\[
\mathbb{E} \left[ \int_T u_t \, dW_t \int_T v_t \, dW_t \right] = \int_T \mathbb{E}[u_t v_t] \, \mu(dt).
\]

2. Let \( W = (W_t)_{t \in [0,1]} \) be a standard Brownian motion. Calculate

\[
\int_0^1 W_t^2 \, dW_t
\]

by using the Itô formula.

3. Let \( W = (W_t)_{t \in [0,1]} \) be a standard Brownian motion. Calculate

\[
\int_0^1 W_t^2 \, dW_t
\]

by interpreting it as multiple Wiener integrals.

4. Consider the stochastic “differential” equation

\[
Y_t = 1 + \int_0^t Y_s \, dW_s,
\]

where \( W \) is a standard Brownian motion. Show that the solution is the stochastic exponent

\[
Y_t = \mathcal{E}(1)_t = e^{W_t - \frac{1}{2} t}.
\]

5. Let \( W \) be the standard Brownian motion on \([0,1]\) and let \( u \in L^2_a([0,1] \times \Omega) \). Let \( \mathcal{E} \) be the stochastic logarithm, i.e. the inverse operator of the stochastic exponent \( \mathcal{E} \):

\[
\mathcal{E}(u)_t = \exp \left( \int_0^t u_s \, dW_s - \frac{1}{2} \int_0^t u_s^2 \, ds \right).
\]

What is the expression for \( \mathcal{E} \)? What is the stochastic “differential” equation that \( \mathcal{E}(u) \) solves?